Re-cap

- Forward vs. Inverse Kinematics
- Robot configuration, Configuration Space
- Vector and Matrix Reviews
- Rotations – Length Preserving Transformations
- Homogeneous Transformations – To describe changes in robot motions
- Constraints: Holonomic vs. Non-holonomic

Dimension of the Configuration Space

Configuration Space:
Set of all possible configurations of the robot

Dimension of the Configuration ==
Number of DOF – Number of Holonomic Constraints

Non-holonomic Constraints: DO NOT constrain position!

Idealized Knife-Edge Constraint

Single contact point at C
Velocity constrained to be along the knife edge
Lateral Velocity = 0

Configuration (pose):
Velocity at point C

Velocity constrained to be along the


Non-Holonomic Constraints

The robot can reach everywhere in the configuration space, BUT, It is under-actuated, and thus the velocity is constrained.

Testing for Integrability of Constraints

Given a constraint in the form of

Let \( v = \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \) then, if \( \nabla \times v = 0 \) constraint can be integrated
**Forward Kinematics of Differential Steering**

Given the robot geometry and wheel speeds, what is the robot’s velocity?

Let:
- \( r \) – wheel radius
- \( l \) – axle length
- \( \dot{\phi}_R \) – right wheel speed
- \( \dot{\phi}_L \) – left wheel speed

**Forward Velocity**

\[
v_f = \frac{r}{2}(\dot{\phi}_R + \dot{\phi}_L)
\]

**Angular Velocity**

\[
\dot{\theta} = \frac{r}{l}(\dot{\phi}_R - \dot{\phi}_L)
\]

**World Coordinates**

**Differential Steering**

Instantaneous Center of Curvature

\[
R_{ICC} = \frac{l}{2} \frac{\dot{\phi}_R + \dot{\phi}_L}{\dot{\phi}_R - \dot{\phi}_L}
\]

**Inverse Kinematics**

\[
\begin{aligned}
\dot{x} &= \frac{r}{2} \cos \theta \frac{\dot{\phi}_L - \dot{\phi}_R}{l} \\
\dot{y} &= \frac{r}{2} \sin \theta \frac{\dot{\phi}_L - \dot{\phi}_R}{l} \\
\dot{\theta} &= \frac{r}{l} (\dot{\phi}_R - \dot{\phi}_L)
\end{aligned}
\]

\[
\dot{\phi}_L = \frac{1}{r} \left( v_f - \frac{1}{2} \dot{\theta} \right)
\]

\[
\dot{\phi}_R = \frac{1}{r} \left( v_f + \frac{1}{2} \dot{\theta} \right)
\]
Differential Steering

- Advantages:
  - Simple construction
  - Zero minimum turning radius (i.e. holonomic!)

- Disadvantages:
  - Small errors in wheel speeds => LARGE position errors
  - Requires 2 drive motors
  - Stability may be an issue

Mobile Robotics

Mobile Robotics

Other Kinematic Models

- Tricycle:
  - Steerable powered front wheel
  - Free spinning rear wheels

Sensing Light

How do we see the world?

Place a piece of film in front of an object, can we get a reasonable image?
Let's add a barrier, with a "VERY" small opening. Purpose: To block off most of the light rays
- Reduces blurring
- The opening is known as aperture
- What happens to the image?

Pinhole Model:
- Captures pencil of rays – all rays through a single point
- This point is called Center of Projection (COP)
- The image is formed on the Image Plane
- Effective focal length, \( f \), is the distance from COP to the Image Plane

"Home-made" Pinhole Camera
www.debevec.org/Pinhole/

Cameras with Lenses
Purpose for the lens:
- Ideal Pinhole Model is unattainable; light gathering mechanism
- Keep image in sharp focus
Anatomy of a modern camera

Sensor Arrays

CMOS sensor

Image Sensors

- Images are formed by the interaction of the incident image irradiance with light sensitive elements on the image plane
- Light sensitive elements
  - Film
  - Charge Coupled Device (CCD)
  - CMOS imaging element

Digital Imaging Systems

- CCD or CMOS imaging array
  - When light falls on the cells in these arrays a charge accumulates which is proportional to the incident light energy
- A/D conversion unit
- Host Computer

Digital Snapshots

- A digital image is an array of numbers indicating the image irradiance at various points on the image
- Image intensities are spatially sampled
- Intensity values are quantized (8-bits, 10-bits, 12-bits, etc)
- Video Imagery
  - For a video camera, images are taken sequentially by opening and close the shutter 30x/sec (i.e. 30 frames/sec)
**Sensing Color**

- In a 3-CCD video camera the light path is split 3 ways which are passed through colored light filters and then imaged.
- As a result – a color image contains three channels of information: red, green, and blue image intensities.
- In a 1-CCD color camera color information is obtained by converting the individual elements with a spatially varying pattern of filters, RGB.

**Bayer pattern used to capture color images on a single imaging surface**

**Practical Color Sensing: Bayer Grid**

Estimate RGB at ‘G’ cells from neighboring values.

http://en.wikipedia.org/wiki/Bayer_filter

**Images in a Computer**

An image is a 2-D table of numbers or 2D Matrix.

$I(i,j)$ is the sensor value at location $x = i$, $y = j$.

$I(2,1) = 134$

$I(3,4) = 136$

Any 2D matrix can be seen as an image.

Example:

$$(r,g,b) = (255,255,251)$$

$$(222,15,7)$$

$$(0,0,0)$$

$$(89,120,1)$$

$$(246,99,0)$$

$$(19,37,87)$$

$$(255,255,115)$$
A word of caution

- For this class, image processing in MatLab
- MatLab extremely powerful in terms of matrix manipulation and signal processing
- Implementation in MatLab vs. in C/C++/Java NOT EQUAL!!

Dimensionality Reduction Machine (3D => 2D)

What have we lost?

- Angles
- Distances (lengths)

Funny things happen ...

Parallel lines are not …

Distances can't be trusted …

Field of View (Zoom)

From London and Upton
Filed of View (Zoom)

From London and Upton

FOV Depends on Focal Length

Size of field of view governed by the size of the image plane:

\[ \varphi = \tan^{-1} \frac{d}{2f} \]

Smaller FOV = larger Focal Length

Conversion from mm to pixels

- In the digital camera, measurements are made in image pixels. Need to convert the focal length (in mm) to pixels.
- Typical digital sensors are smaller than (35 mm) film, this effectively increases the focal length.

From Zisserman & Hartley
**Field of View/Focal Length**

- **Large FOV, small f**
  - Camera closer to car

- **Small FOV, large f**
  - Camera far from car

**Large Focal Length Compresses Depth**

<table>
<thead>
<tr>
<th>400 mm</th>
<th>200 mm</th>
<th>100 mm</th>
<th>50 mm</th>
<th>28 mm</th>
<th>17 mm</th>
</tr>
</thead>
</table>

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**Image Projection**

Modeling (pinhole) projection

- The coordinate system
  - Use the pinhole model as an approximation
  - Put the optical center (COP) at origin
  - Put image plane (Projection Plane, PP) in front of the COP
  - Why?
  - The camera looks down the negative z-axis
  - Required for right-handed coordinates

**Projection Equations**

Note: \(\overrightarrow{OP'}\) and \(\overrightarrow{OP}\) are colinear

Given \[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]
then
\[
x' = \frac{fx}{z} \\
y' = \frac{fy}{z}
\]