MEM355 Performance Enhancement of Dynamic Systems
Spring 2012
Professor: Ani Hsieh
Time: Tues, Thurs 3:30-4:50pm
Location: 231 Curtis Hall

General Guidelines
- Course website: http://robotics.mem.drexel.edu/mhsieh/Courses/MEM355
- Textbook:
  - Main Text: Control Systems Engineering 5th Ed by Norman Nise
- Grading:
  - Homework 30%, Participation 10%, Exams 60%
  - Weekly Quizzes
  - 3 Exams: Weeks 3, 7, and Finals Exam Week
  - Seniors: please send email to TA to make final exam arrangements
- Homework is due every Tuesday at the end of class. No exceptions!
- Late submissions WILL NOT be accepted!
- Participation
  - In-class and during recitation, attendance
  - Peer help – w/ homework, online discussion board and chat sessions
- Participation Evaluations

Modeling of Electrical Networks
Conservation Laws
1. Kirchhoff’s Voltage Law
   Sum of voltages around a closed path is zero.
2. Kirchhoff’s Current Law
   Sum of current into a node equals zero.

Modeling of Mechanical Systems
Conservation Laws
1. Newton’s Second Law / D’Alembert’s Principle
   \( F = ma \)
2. Conservation of Linear/Angular Momentum
   \( \frac{d}{dt} \mathbf{x} = \mathbf{F} \) or \( \frac{d}{dt} \mathbf{L} = \mathbf{T} \)
3. Conservation of Energy
   \( (\text{Potential} + \text{Kinetic})_{\text{Initial}} = (\text{Potential} + \text{Kinetic})_{\text{Final}} \)
Analogous Quantities
- Resistance (R) <-> Lubricity (1/f_o)
- Capacitance (C) <-> Mass (M)
- Inductance (L) <-> Compliance (1/K)

Analogous Equations (Elec <-> Mech)
- \( i(t) = \frac{v(t)}{R} \) <-> \( F = f v(t) \)
- \( i(t) = \frac{1}{L} \int v(t) \, dt \) <-> \( F = M \frac{dv(t)}{dt} = M a(t) \)
- \( i(t) = \frac{1}{C} \int i(t) \, dt \) <-> \( F = K \int v(t) \, dt = K x(t) \)

An Example
A 200 kg lander is approaching Mars. Assume a gravitational acceleration of 3.7 m/s^2. Given an initial height of 15 m and velocity of 2.5 m/s. What is the average thrust needed to land w/ a touchdown speed of 0.5 m/s after 10s?

\[
\int_{0}^{t_f} (-mg_M + T) \, dt = m \left( -v_f - (-v_i) \right)
\]
\[
(-mg_M + T) \int_{0}^{t_f} dt = m \left( v_i - v_f \right)
\]
\[
\left( -3.7 + \frac{T}{200} \right) = 2
\]
\[
T = -0.0175 \text{ (N)}
\]
What if T(t)?

A 200 kg lander is approaching Mars. Assume a gravitational acceleration of $3.7 \text{ m/s}^2$. Given an initial height of 15 m and velocity of 2.5 m/s. What is the average thrust needed to land with a touchdown speed of 0.5 m/s after 10s?

\[
\int F \, dt = \Delta L
\]
\[
\int_0^t (-mg_M + T) \, dt = m \left( -v_f - (-v_i) \right)
\]
\[
\frac{d}{dt} \left[ \int_0^t (-mg_M + T) \, dt \right] = \frac{d}{dt} \left( m(v_i - v_f) \right)
\]
\[
-mgM + T(t) = \dot{m} \dot{v}
\]
\[
\ddot{x} = T(t) - mg
\]

Review I – Laplace Transform

Why Laplace Transforms?

- Analyze systems described by differential equations
- Laplace transforms turns Differential Equations $\Rightarrow$ Algebraic Equations
- Enables representation of system dynamics using pictures

Laplace Transforms

Integral Transform – Maps functions from the time domain to the frequency domain

\[
\mathcal{L} \{ f(t) \} = \int_0^\infty f(t) e^{-st} \, dt
\]

with

\[
s = \sigma + j\omega
\]

Laplace Transforms are extremely useful when analyzing Linear Ordinary Differential Equations.

1. Transforms differential equations into algebraic eqns.
2. Allows us to be able to draw block diagrams from the differential eqns.
3. Trade-off we have to work in the frequency domain
Inverse Laplace Transforms

Integral Transform – Maps functions from the frequency domain to the time domain

\[ \mathcal{L}^{-1} [F(s)] = \frac{1}{2\pi j} \int_{\sigma-j\omega}^{\sigma+j\omega} F(s)e^{st}dt \]

Therefore,
1. Convert \( f(t) \) to \( F(s) \)
2. Analyze \( F(s) \).
3. Convert back.
4. Generally – this results in obtaining the time domain solution to the differential equation.

Partial Fraction Expansion

To find the inverse Laplace Transform of some function \( F(s) \), we must have the ability to “factorize” fractions of polynomials

\[ F(s) = \frac{s^3 + 2s^2 + 6s + 7}{s^2 + s + 5} \]

Now what?

\[ F(s) = s + 1 + \frac{2}{s^2 + s + 5} \]
Summary 1

(Inverse) Laplace Transforms

time domain <-> frequency domain
differential equation <-> algebraic equation

Partial Fraction Expansion

factorizes “complicated” $F(s)$ expressions $L^{-1}[F(s)]$
-> significantly simplifies ability to take

Transfer Function

$v(t) = L\frac{di}{dt} + Ri(t) + \frac{1}{C} \int i(t) dt$

$v(t) = L\frac{d^2 x}{dt^2} + R\frac{dx}{dt} + \frac{1}{C} q(t)$

$f(t) = M\frac{d^2 x}{dt^2} + f_v\frac{dx}{dt} + kx(t)$

Purpose: Relate a system’s output to its input

1. Easy separation of INPUT, OUTPUT, SYSTEM (PLANT)
2. Algebraic relationships (vs. differential)
3. Easy interconnection of subsystems in a MATHEMATICAL framework

A General N-Order Linear, Time Invariant ODE

$\sum_{i=0}^{n} a_i \frac{d^i c(t)}{dt^i} \frac{d^{n-1} c(t)}{dt^{n-1}} \ldots \frac{d c(t)}{dt} + \sum_{j=1}^{m} b_j \frac{d^j r(t)}{dt^j} \frac{d^{m-1} r(t)}{dt^{m-1}} \ldots \frac{d r(t)}{dt} = 0$

Given: $c(t)$ – output $r(t)$ – input

Furthermore, if we know $G(s)$, then

output = $G(s) \times$ input

And, if we can take $C^{-1}[G(s)] \times$ input we get ….
All Together Now

Example:

\[ v(t) = L \frac{di}{dt} + Ri(t) + \frac{1}{C} \int i(t) \, dt \]

\[ v(t) = L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q(t) \]

Underlying Assumptions

Linearly

1. Superposition
   \[ f(x_1 + x_2) = f(x_1) + f(x_2) \]

   \[ c_1(t) + c_2(t) \]
   \[ r_1(t) + r_2(t) \]

   System

2. Homogeneity
   \[ f(\alpha x) = \alpha f(x) \]

   \[ a c_1(t) \]
   \[ a r(t) \]

Because the Laplace Transform is Linear!

Intermission: State-Space Representation

Alternate approach to the Transfer Function

Time domain (vs. frequency domain)

Allows for multiple inputs and/or outputs

Versatility – our initial conditions DO NOT have to be 0

\[ \frac{d^n c(t)}{dt^n} + h_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \ldots + h_0 c(t) = h_{n-r} \frac{d^n r(t)}{dt^n} + h_{n-1-r} \frac{d^{n-1} r(t)}{dt^{n-1}} + \ldots + h_0 r(t) \]

Converts N-th order differential equation into N simultaneous FIRST-ORDER differential equations

State-Space Representation
Review III – Block Diagrams

Block Diagrams
- Schematic of the system (vs. mathematical description)
- Visualize the flow of the various signals within a system
- Modular design
- System Integration

Elements of Block Diagrams
- Signals
- System/Plant
- Summing Junctions
- Pickoff Points

Common Interconnection Topologies
- Cascade Form

\[ R(s) \rightarrow G_1(s) \rightarrow G_2(s) \rightarrow G_3(s) \rightarrow C(s) \]
Common Interconnections
Topologies

Parallel Form

Feedback Form

Positive or Negative Feedback

Equivalent Forms

Equivalent Forms
**Block Diagram Reduction – Example 1**

- Input: \( R(s) \)
- Output: \( C(s) \)
- Block Diagram Reduction:
  - \( G_1(s) \), \( H_1(s) \)
  - \( G_2(s) \), \( H_2(s) \)

Answer:

\[
\frac{R(s) G_1(s) G_2(s)}{1 + G_1(s) G_2(s) H_1(s) - H_1(s) + H_2(s)}
\]

**Block Diagram Reduction – Example 2**

- Input: \( R(s) \)
- Output: \( C(s) \)
- Block Diagram Reduction:
  - \( x \)
  - \( \frac{1}{s^2} \)
  - \( F(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s} \)

**Review IV – 1st Order System Response**

- From Differential Equation:
  - Output Response = Particular + Homogeneous
    - Forced/Steady-State Response
    - Natural Response
- So far, solving differential equations:
  - Brute Force
  - Laplace Transform
- Question:
  - Can we get a qualitative (rather than exact) solution for system response?
  - Why?
Poles
- Given a transfer function: \( G(s) = \frac{N(s)}{D(s)} \)
- Definition: \( \{s \mid G(s) = \infty \text{ and } D(s) = 0 \text{ s.t. } N(s) = 0\} \)
- Example: \( G(s) = \frac{s + 2}{s(s + 5)} \)
- Conventionally, given \( G(s) = \frac{s^2 + 2s + 1}{s(s + 1)(s + 5)} \)
  \( s = -1 \) is a pole, even though \( G(-1) \neq \infty \)

Zeros
- Given a transfer function: \( G(s) = \frac{N(s)}{D(s)} \)
- Definition: \( \{s \mid G(s) \neq 0 \text{ and } N(s) = 0 \text{ s.t. } D(s) = 0\} \)
- Example: \( G(s) = \frac{s + 2}{s(s + 5)} \)
- Conventionally, given \( G(s) = \frac{(s + 1)(s + 2)}{s(s + 1)(s + 5)} \)
  \( s = -1 \) is a zero, even though \( G(-1) \neq 0 \)

The s-plane
- Recall, \( s = \sigma + j\omega \)
- Zeros – \( \bigcirc \)
- Poles – \( x \)

First Order Systems
- In general, given \( G(s) = \frac{\frac{s + b}{s + a}}{s + n} \)
- Let \( R(s) = \frac{1}{s} \), then \( C(s) = \frac{s + b}{s(s + a)} \)
  \( K_1 = \frac{b}{a} \)
  \( K_2 = 1 - \frac{b}{a} \)
- As such, \( c(t) = K_1 e^{-\alpha t} + K_2 e^{-\beta t} \)
- Therefore, \( c(t) = \frac{b}{a} + (1 - \frac{b}{a}) e^{-\alpha t} \)
First Order Systems

- Given, \( G(s) = \frac{s+1}{s+\alpha} \) with \( R(s) = \frac{1}{s} \), results in
  \[ c(t) = \frac{\beta}{\alpha} + \left(1 - \frac{\beta}{\alpha}\right)e^{-\alpha t} \]

- Note the following:
  1. Pole of \( R(s) \) generates the form of the forced response
  2. Pole of \( G(s) \) generates the form of the natural response
  3. Pole on the real axis generates an exponential response of the form \( e^{-\alpha t} \), where \(-\alpha\) is the pole location on the axis

4. Zeros and poles generate the amplitudes of the responses

Obtaining \( G(s) \) Empirically

- Assume \( G(s) = \frac{K}{s^2 + \omega^2} \) has a step response of
  \[ C(s) = \frac{K}{s^2 + \omega^2} = \frac{K}{s} - \frac{K}{\omega} \]

- If the signal looks like a 1st order response, then obtain \( K \) and \( \alpha \) by measuring \( T_0, T_r, T_s \) and response amplitude

Characterizing First – Order Systems

- Given \( G(s) = \frac{\alpha}{s + \alpha} \) with \( R(s) = \frac{1}{s} \)
  Then \( c(t) = \alpha(1 - e^{-\alpha t}) \)

- Time Constant:
  - Time for \( e^{-\alpha t} \) to decay 37% of its initial value, \( T_c = 1/\alpha \)
- Rise Time:
  - Time for the signal to go from 0.1 to 0.9 of its final value, \( T_r = 2.2/\alpha \)
- Settling Time:
  - Time for the signal to reach & stay within 2% of its final value, \( T_s = 4/\alpha \)