MEM355 Performance Enhancement of Dynamic Systems  
Spring 2012

Professor: Ani Hsieh  
Time: Tues, Thurs 3:30-4:50pm  
Location: 231 Curtis Hall

General Guidelines
- Course website: http://robotics.mem.drexel.edu/mhsieh/Courses/MEM355  
- Textbook:  
  Main Text: Control Systems Engineering 5th Ed by Norman Nise  
- Grading:  
  - Homework 30%, Participation 10%, Exams 60%  
  - Weekly Quizzes  
  - 3 Exams: Weeks 3, 7, and Finals Exam Week  
  - Seniors: please send email to TA to make final exam arrangements  
  - Homework is due every Tuesday at the end of class. No exceptions!  
  - Late submissions WILL NOT be accepted!  
- Participation  
  - In-class and during recitation, attendance  
  - Peer help – w/ homework, online discussion board and chat sessions  
- Participation Evaluations

An Example
A 200 kg lander is approaching Mars. Assume a gravitational acceleration of 3.7 m/s^2. Given an initial height of 15 m and velocity of 2.5 m/s. What is the average thrust needed to land with a touchdown speed of 0.5 m/s after 10s?

\[
\int F \, dt = \Delta L
\]
\[
\int_{0}^{T} \left(-mg_M + T\right) \, dt = m \left(v_f - (v_i)\right)
\]
\[
\left(-mg_M + T\right) \int_{0}^{T} \, dt = m \left(v_i - v_f\right)
\]
\[
\left(-3.7 + \frac{T}{200}\right) \times 2 = 2
\]
\[
T = -0.0175 \text{ (N)}
\]

What if T(t)?
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\int F \, dt = \Delta L
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\[
\int_{0}^{T} \left(-mg_M + T\right) \, dt = m \left(v_f - (v_i)\right)
\]
\[
\frac{d}{dt} \int_{0}^{T} \left(-mg_M + T\right) \, dt = \frac{d}{dt} \left(m(v_i - v_f)\right)
\]
\[
-mg_M + T(t) = m\dot{v}
\]
\[
m\ddot{x} = T(t) - mg
\]
Review I – Laplace Transform

Summary 1
(Inverse) Laplace Transforms
- time domain <-> frequency domain
- differential equation <-> algebraic equation

Partial Fraction Expansion
- factorizes "complicated" F(s) expressions $L^{-1}[F(s)]$
- significantly simplifies ability to take

Review II – Transfer Functions

Transfer Function
- $v(t) = L\frac{di}{dt} + Ri(t) + \frac{1}{C} \int i(t) dt$
- $v(t) = L\frac{dq}{dt} + Rd\frac{dq}{dt} + \frac{1}{C} q(t)$
- $f(t) = M\frac{dx}{dt^2} + f\frac{dx}{dt} + kx(t)$

Purpose: Relate a system’s output to its input
1. Easy separation of INPUT, OUTPUT, SYSTEM (PLANT)
2. Algebraic relationships (vs. differential)
3. Easy interconnection of subsystems in a MATHEMATICAL framework
A General N-Order Linear, Time Invariant ODE

\[ a_n \frac{d^n c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \ldots + a_1 \frac{d c(t)}{dt} + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \ldots + b_0 r(t) \]

Given: \( c(t) \) – output
\( r(t) \) – input
Furthermore, if we know \( G(s) \), then
output = \( G(s) \times \) input
And, if we can take \( C^{-1} \{ G(s) \times \) input \} we get ….

Intermission: State-Space Representation

State-Space Representation

- Alternate approach to the Transfer Function
- Time domain (vs. frequency domain)
- Allows for multiple inputs and/or outputs
- Versatility – our initial conditions DO NOT have to be 0
- Converts N-th order differential equation into N simultaneous FIRST-ORDER differential equations

Review III – Block Diagrams
**Block Diagrams**
- Schematic of the system (vs. mathematical description)
- Visualize the flow of the various signals within a system
- Modular design
- System Integration

**Block Diagram Reduction – Example 2**

```
Answer: \( F(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s} \)
```

**Review IV – 1st Order System Response**

- From Differential Equation:
  - Output Response = Particular + Homogeneous
  - Forced/Steady-State Response
  - Natural Response
- So far, solving differential equations:
  - Brute Force
  - Laplace Transform
- Question:
  - Can we get a qualitative (rather than exact) solution for system response?
  - Why?
### Poles
- Given a transfer function: \( G(s) = \frac{N(s)}{D(s)} \)
- Definition: \( \{ s \mid G(s) = \infty \text{ and } D(s) = 0 \text{ s.t. } N(s) = 0 \} \)
- Example: \( G(s) = \frac{s + 2}{s(s + 5)} \)
- Conventionally, given \( G(s) = \frac{s^2 + 2s + 1}{s(s + 1)(s + 5)} \)

### Zeros
- Given a transfer function: \( G(s) = \frac{N(s)}{D(s)} \)
- Definition: \( \{ s \mid G(s) = 0 \text{ and } N(s) = 0 \text{ s.t. } D(s) = 0 \} \)
- Example: \( G(s) = \frac{s + 2}{s(s + 5)} \)
- Conventionally, given \( G(s) = \frac{(s + 1)(s + 2)}{s(s + 1)(s + 5)} \)

### The s-plane
- Recall, \( s = \sigma + j\omega \)
- Zeros – \( \infty \)
- Poles – \( x \)

### First Order Systems
- In general, given \( G(s) = \frac{s + b}{s + a} \)
- Let \( R(s) = 1/s \), then \( C(s) = \frac{s + b}{s(s + a)} = \frac{K_1}{s + a} \frac{K_2}{s} \)
- As such, \( c(t) = K_1 + K_2 e^{-at} \)
- \( K_1 = \frac{b}{a} \)
- \( K_2 = 1 - \frac{b}{a} \)
- Therefore, \( c(t) = \frac{b}{a} (1 - \frac{b}{a}) e^{-at} \)
First Order Systems

- Given $G(s) = \frac{s + 1}{s + \alpha}$ with $R(s) = 1/s$, results in
  $c(t) = \frac{1}{\alpha} (1 - e^{-\alpha t})$

- Note the following:
  1. Pole of $R(s)$ generates the form of the forced response
  2. Pole of $G(s)$ generates the form of the natural response
  3. Pole on the real axis generates an exponential response of the form $e^{-\alpha t}$, where $-\alpha$ is the pole location on the axis
  4. Zeros and poles generate the amplitudes of the responses

Example

- Given $G(s) = \frac{10(s + 4)(s + 6)}{(s + 1)(s + 7)(s + 8)(s + 10)}$
- Poles? Zeros?
- What if $R(s) = 1/s$? What happens to the solution?

Characterizing First – Order Systems

- Given $G(s) = K$ with $R(s) = 1/s$
  Then $c(t) = K e^{-\alpha t}$

- Time Constant:
  - Time for $e^{-\alpha t}$ to decay 37% of its initial value, $T_c = 1/\alpha$
- Rise Time:
  - Time for the signal to go from 0.1 to 0.9 of its final value, $T_r = 2.2/\alpha$
- Settling Time:
  - Time for the signal to reach & stay within 2% of its final value, $T_s = 4/\alpha$

Obtaining $G(s)$ Empirically

- Assume $G(s) = \frac{K}{s + \alpha}$ has a step response of
  $C(s) = K e^{-\alpha s}$
- If the signal looks like a 1st order response, then obtain $K$ and $\alpha$ by measuring $T_c$, $T_r$, $T_s$, and response amplitude